



Roll No.

ANNA UNIVERSITY (UNIVERSITY DEPARTMENTS)

B.E. / B. Tech / B. Arch (Full Time) - END SEMESTER EXAMINATIONS, APR/MAY 2024

INFORMATION TECHNOLOGY

Fourth Semester (IV)

IT5402 & Design and Analysis of Algorithms

(Regulation 2019)

Time: 3hrs

Max. Marks: 100

CO 1	Articulate the process of problem solving and writing algorithms
CO 2	Understand different algorithmic design strategies
CO 3	Design and implement any problem using design techniques
CO 4	Critically analyse the complexity of the given problem.
CO 5	Solve a problem in polynomial time or prove that to be an NP-Complete problem.
CO 6	Obtain knowledge of advanced topics such as approximation algorithms, linear programming and randomized algorithms.

BL – Bloom's Taxonomy Levels

(L1 - Remembering, L2 - Understanding, L3 - Applying, L4 - Analysing, L5 - Evaluating, L6 - Creating)

PART- A (10 x 2 = 20 Marks)
(Answer all Questions)

Q. No	Questions	Marks	CO	BL																		
1	Compare the growth of functions where $f(n)=\log_2 n$ and $g(n) = \log_3 n$	2	<u>CO4</u>	<u>L3</u>																		
2	Prove that $\frac{1}{8}n^2 - \frac{3}{2}n = \Theta(n^2)$ by finding positive constants for c_1 , c_2 and n_0 .	2	<u>CO4</u>	<u>L3</u>																		
3	Derive the worst case analysis of Merge sort algorithm from its recurrence relation.	2	<u>CO4</u>	<u>L2</u>																		
4	Write down the Memoized Matrix chain procedure which is used to resolve overlapping sub problems.	2	<u>CO2</u>	<u>L2</u>																		
5	Consider an activity selection problem with start time and end time as follows: Activity 1 2 3 4 5 6 7 8 Start Time 1 4 2 6 3 7 9 11 End Time 2 7 5 9 6 10 11 14 Find out the optimal set of activities to be completed using Greedy strategy.	2	<u>CO3</u>	<u>L2</u>																		
6	Find the maximum profit using greedy strategy by finding the subset of items to be added in the knapsack whose capacity is 150 kg.	2	<u>CO3</u>	<u>L3</u>																		
	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th>Items</th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> </tr> </thead> <tbody> <tr> <td>Weights (in kg)</td> <td>15</td> <td>25</td> <td>30</td> <td>55</td> <td>60</td> </tr> <tr> <td>Value</td> <td>30</td> <td>40</td> <td>100</td> <td>78</td> <td>90</td> </tr> </tbody> </table>	Items	1	2	3	4	5	Weights (in kg)	15	25	30	55	60	Value	30	40	100	78	90			
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Weights (in kg)	15	25	30	55	60																	
Value	30	40	100	78	90																	
7	Write down the Schur complement of A with respect to a_{11} . Also Prove that $A = LU$.	2	<u>CO2</u>	<u>L2</u>																		
8	Convert the following Linear program into a standard form: Minimize $-4x_1 + 3x_2 - 4x_3$	2	<u>CO6</u>	<u>L3</u>																		

	Subject to the constraints $x_1 - x_2 = 8$ $2x_1 + x_2 + x_3 \leq 7$ $-5x_1 - 2x_2 + 3x_3 \geq -2$ $x_1, x_2 \geq 0$			
9	Define the problems P, NP and NPC.	2	CO5	<u>L2</u>
10	Write down the verification algorithm A which verifies the language L belongs to NP.	2	CO5	<u>L2</u>

PART- B (5 x 13 = 65 Marks)
(Restrict to a maximum of 2 subdivisions)

Q. No	Questions	Marks	CO	BL
11 (a)(i)	Solve the following Recurrence relation using Recursion Tree approach: $T(n) = 3T(n/4) + cn^2$	9	<u>CO1</u> <u>CO4</u>	<u>L3</u>
(ii)	Using Masters Theorem, solve the following recurrence relation: $T(n) = 9T(n/4) + n^2$	4	<u>CO1</u> <u>CO4</u>	<u>L2</u>
OR				
11 (b) (i)	Given an array of size n and a number k, find all the elements that appear more than n/k times with a suitable algorithm. Derive the recurrence relation for the algorithm and solve the time complexity using Backward Substitution method.	9	<u>CO1</u> <u>CO4</u>	<u>L3</u>
(ii)	Write down the Insertion Sort algorithm and determine the operation count of the algorithm to derive its worst case time complexity.	4	<u>CO1</u> <u>CO4</u>	<u>L2</u>
12 (a) (i)	Write down the Quick Sort algorithm using Divide and Conquer approach and derive the best case, worst case and average case analysis of the algorithm from the recurrence relation formed for the respective cases. Solve the recurrence relation using Recursion Tree method	9	<u>CO3</u>	<u>L3</u>
(ii)	Simulate the steps involved in editing the distance between the strings CLEVER to RIVER.	4	<u>CO3</u>	<u>L3</u>
OR				
12 (b)	Define the problem solving strategy used to determine the order for multiplying a set of matrices at a lowest cost. i) Write down the steps involved in that strategy. ii) Compute and Construct the optimal solution to multiply the following set of matrices (at a minimum cost) by writing necessary algorithms: A1 (6 * 5) A2 (5 * 4) A3 (4 * 7) A4 (7 * 3) (Simulate the steps with the necessary tables in a detailed manner)	13	<u>CO3</u>	<u>L3</u>
13 (a)	Find the L, U and P values for the following set of linear equations. Solve the equations to determine the values of x_1, x_2 and x_3 after	13	CO2	<u>L3</u> <u>L2</u>

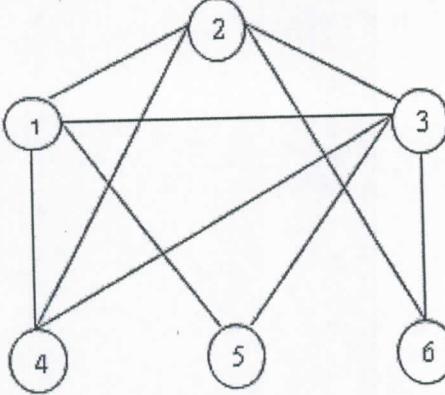


	decomposition. Write down the algorithms involved in LUP decomposition process.			
	$x_1 + x_2 - x_3 = 4$			
	$x_1 - 2x_2 + 3x_3 = -6$			
	$2x_1 + 3x_2 + x_3 = 7$			

OR

13 (b) (i)	<p>A text document consisting of a set of characters with its repeated occurrence is shown below. Compress the document efficiently using Huffman coding technique. Simulate the compression process step by step.</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td>A</td><td>X</td><td>T</td><td>B</td><td>U</td><td>F</td><td>H</td></tr> <tr> <td>150</td><td>225</td><td>247</td><td>382</td><td>215</td><td>230</td><td>347</td></tr> </table> <p>Write down the algorithm involved in generating Huffman code for the above text document. Represent the string "TBUIFX" using generated code.</p>	A	X	T	B	U	F	H	150	225	247	382	215	230	347	9	CO2	<u>L3</u>
A	X	T	B	U	F	H												
150	225	247	382	215	230	347												
(ii)	<p>Find an optimal set cover for the below problem using greedy strategy with a suitable algorithm:</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td>$S_1 = \{4, 5, 8, 9\}$</td> <td>$\text{cost}(S_1) = 10$</td> </tr> <tr> <td>$S_2 = \{2, 4, 6, 8, 10\}$</td> <td>$\text{cost}(S_2) = 20$</td> </tr> <tr> <td>$S_3 = \{1, 2, 4, 6, 7, 9, 10, 11\}$</td> <td>$\text{cost}(S_3) = 15$</td> </tr> <tr> <td>$S_4 = \{3, 8, 10\}$</td> <td>$\text{cost}(S_4) = 11$</td> </tr> <tr> <td>$S_5 = \{1, 6, 7, 13\}$</td> <td>$\text{cost}(S_5) = 14$</td> </tr> </table>	$S_1 = \{4, 5, 8, 9\}$	$\text{cost}(S_1) = 10$	$S_2 = \{2, 4, 6, 8, 10\}$	$\text{cost}(S_2) = 20$	$S_3 = \{1, 2, 4, 6, 7, 9, 10, 11\}$	$\text{cost}(S_3) = 15$	$S_4 = \{3, 8, 10\}$	$\text{cost}(S_4) = 11$	$S_5 = \{1, 6, 7, 13\}$	$\text{cost}(S_5) = 14$	4	CO2	<u>L2</u>				
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$S_5 = \{1, 6, 7, 13\}$	$\text{cost}(S_5) = 14$																	
14 (a) (i)	<p>Solve the following linear equations using Simplex method</p> <p>Maximize $300x_1 + 350x_2$ Subject to the constraints $x_1 + 3x_2 \leq 24$ $3x_1 + 2x_2 \leq 36$ $3x_1 + 4x_2 \leq 44$ $x_1, x_2 \geq 0$</p>	9	CO6	<u>L3</u>														
(ii)	<p>Write down the Pivoting and Simplex algorithms involved in solving the linear equations.</p>	4	CO6	<u>L2</u>														
14 (b) (i)	<p>Determine whether the initial basic feasible solution is possible for the following linear equations:</p> <p>Maximize $x_1 - 2x_2$ Subject to the constraints: $2x_1 - 3x_2 \leq 4$ $x_1 - 3x_2 \leq -6$ $x_1, x_2 \geq 0$ If basic feasible solution is available, find the appropriate objective function and slack form for the above equations. Also write down the necessary algorithm involved in this process.</p>	9	CO6	<u>L3</u>														



(ii)	<p>Write down the steps involved in converting a primal linear problem to a dual linear problem. Convert the following primal problem to dual:</p> <p>Maximize $6x_1+x_2+x_3$ Subject to the constraints $9x_1-x_2+x_3 \leq 18$ $24x_1+x_2-4x_3 \leq 42$ $12x_1-3x_2+4x_3 \leq 96$ $x_1, x_2, x_3 \geq 0$</p>	4	CO6	L2
15 (a)	<p>Write down the properties of NP Complete. Prove that satisfiability of Boolean formulas in 3 Conjunctive Normal form (3-CNF) is NP Complete in a detailed manner.</p>	13	CO5	L3
OR				
15 (b) (i)	<p>What is a clique problem? How do you convert the Clique optimization problem into a decision problem?</p> <p>Prove that Clique belongs to NPC by reducing from the 3-CNF formula: $\emptyset = C_1 \wedge C_2 \wedge C_3$ where $C_1 = (x_1 \vee \neg x_2 \vee \neg x_3)$ and $C_2 = (\neg x_1 \vee x_2 \vee x_3)$ and $C_3 = (x_1 \vee x_2 \vee x_3)$</p>	9	CO5	L3
(ii)	 <p>Determine the Vertex Cover for the above graph using Approximation algorithm. Also write down the algorithm involved in finding the near optimal solution.</p>	4	CO5 CO6	L3

PART- C (1 x 15 = 15 Marks)
 (Q.No.16 is compulsory)

Q. No	Questions	Marks	CO	BL
16. (i)	<p>Sequence of strings generated by two Large Language models compete each other for similarity of characters among them. String generated by the first LLM is "GTXABAABBTX" whereas the string generated by the second LLM is "XABBTX". How do you compute the similarity of the strings generated by these language models?</p> <p>a) Write down the recurrence relation involved in it. b) Simulate the steps involved in similarity computation in a detailed manner.</p>	10	CO2 CO3	L3



	c) Write down the necessary algorithm involved in this process.			
(ii)	A farmer has 20 hectares for growing barley and rice. The farmer has to decide how much of each to grow. The cost per hectare for barley is Rs.300 and for rice is Rs.150. The farmer has budgeted Rs.980. Barley requires 1-man day per hectare and rice requires 2-man days per hectare as labor. There are 36 man-days available. The profit on barley is Rs.100 per hectare and on rice is Rs.120 per hectare. Find the number of hectares of each crop the farmer should sow to maximize the profits. Tabulate the problem and formulate an LPP to maximize the profit.	5	CO6	L4

